CIE-2 Paper Ch3 Qs Pg28

1) a) Consider a D.E : $a_0 y'' + a_1 y' + a_2 y = 0 = f(x)$, To find the particular solution of this D.E we need some conditions, when initial conditions such as $y(0) = y_1 \rightarrow y'(0) = y_2$, The problem is known as IVP.

c) $(D^3)\,(D^3 - 3D^2 + 3D - 1)\,y = e^x + 2e^{-x} + 4e^{0x}$

$CF = m^3 - 3m^2 + 3m - 1 = 0$

$m_1 = 1, m_2 = 1, m_3 = 1$

$CF = e^x (c_1 + x c_2 + x^2 c_3)$

$$PI = \frac{e^x}{D^3 - 3D^2 + 3D - 1}\Big|_{D=1} + \frac{2e^{-x}}{D^3 - 3D^2 + 3D - 1}\Big|_{D=-1} + \frac{4e^{0x}}{D^3 - 3D^2 + 3D - 1}\Big|_{D=0}$$

$$= \frac{e^x}{1 - 3 + 3 - 1} + \frac{2e^{-x}}{-1 - 3 + 3 - 1} + \frac{4e^{0x}}{-1}$$

$$= \frac{x e^x}{3D^2 - 6D + 3 = 0} - \frac{2e^{-x}}{8} - 4$$

$$= \frac{x^2 e^x}{6D - 6(0)} - \frac{e^{-x}}{4} - 4$$

$$= \frac{x^3 e^x}{6} - \frac{e^{-x}}{4} - 4$$

$y(x) = CF + PI$

d) $y'' - 2y' + y = x$

$(m^2 - 2m + 1) = 0$

$m = 1, 1$

$CF = e^x (c_1 + x c_2)$

$PI =$

$$1 - 2D - D^2 \;\big|\; x \quad (x + 2)$$

$-x - 2$

$0 + 2$

-2

0

$y(x) = e^x (c_1 + x c_2) + x + 2$

$0 = e^0 (c_1) + 2 \qquad [y(0) = 0)$

$0 = c_1 + 2$

$c_1 = -2$

$y(1) = 3$

$3 = e^3 (c_1 + 3c_2) + 5$

$3 = e^3 (-2 + 3c_2) + 5$

$-2 = -2e^3 + 3e^3 c_2$

$\frac{-2 + 2e^3}{3e^3} = c_2$

$0.633 = c_2$

$y = e^x (-2 + x 0.633) + x + 2$

Pg 29.

a) D → differential operator $= \frac{d}{dx}$

$DX = \frac{dX}{dx}$

$\frac{1}{D} \Rightarrow$ Integral

$\frac{1}{D} X = \int X(x)\, dx$

iii) $(D^2 - 4D + 13)\, y = e^x \cosh 2x + 2^x$

$m^2 - 4m + 13 = 0$

$m = 2 \pm 3i$

CF $\Rightarrow e^{2x} (c_1 \cos 3x + c_2 \sin 3x)$

$e^x \cdot \frac{e^{2x} + e^{-2x}}{2} + e^{x \ln 2}$

$\frac{e^{3x} - e^{-x}}{2} + e^{x \ln 2}$

$\left. \frac{e^{3x}}{2D^2 - 8D + 26} \right|_{D=3} - \left. \frac{e^{-x}}{2(D^2 - 4D + 13)} \right|_{D=-1} + \left. \frac{e^{x \ln 2}}{D^2 - 4D + 13} \right|_{D=\ln 2}$

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$$= \frac{e^{3x}}{28} - \frac{e^{-x}}{36} + \frac{e^{x \ln 2}}{10 \cdot 7}$$

$y(x) = CF + PI$.

2 d) Homo:

$m^3 - m^2 + 100m - 100 = 0$

$m = 1, \pm 10i$

$CF = c_1 e^x + c_2 \cos(10x) + c_3 \sin(10x)$

~~$y = c_1 e^0 + c_2 \cos(10) + c_3 \sin(10)$~~

$y' = c_1 e^x + c_2 \sin(10x) \cdot 10 + c_3 \cos(10x) \cdot 10$

$0 = c_1 e'' - c_2 \sin(110) 10 + c_3 \cos(110x) \cdot 10$

$0 = c_1 e'' - 10 c_2 \sin(110) + 10 c_3 \cos(110x)$

~~$y'' = c_1 e^x - c_2 \cos(10x) 100 + c_3 \sin(10x)$~~

$y = c_1 e^x + c_2 \cos(10x) + c_3 \sin(10x)$ — ①

$y' = c_1 e^x + c_2 10 \sin(10x) + c_3 10 \cos(10x)$ — ②

$y'' = c_1 e^x - c_2 100 \cos(10x) + c_3 100 \sin(10x)$ — ③

① $y = c_1 + c_2$

② $11 = c_1 + 10 c_3$

③ $-299 = c_1 - 100 c_2$

$C_1 = 4 - C_2$;

$4 - C_2 - 100C_2 = -2019$

$+101C_2 = +303$

$C_2 = \frac{303}{101} = 3$

$C_1 = 1$

$C_3 = 1$

$y = e^x + 3\cos(10x) + \sin(10x)$.

3 b) $x^2y'' + 9xy' + 25y = 0$

$(x^2D^2 + 9xD + 25)y = 0$

$(D'(D'-1) + 9D' + 25) = 0$

$D'^2 - D' + 9D + 25$

$= D'^2 + 8D + 25 = 0$

$m^2 + 8m + 25 = 0$

$m = -4 \pm 3i$

$CF = e^{-4x}(C_1 \cos 3x + C_2 \sin 3x)$

(1)
(2)
(3)

c) $(x^3D^3 + 3x^2D^2 + xD + 1)\ y = e^x \cdot z$

$D'(D'-1)(D'-2) + 3D'(D'-1) + D' + 1 = e^x z$

$(D''^2 - D')(D'-2) + 3D''^2 - 3D' + D' + 1$

$D''^3 - 2D''^2 - D'' + 2D' + 3D'' - 3D' + D'$

$(D''^3 + 1) = 0$

$m^3 + 1 = 0$

$m^3 = -1$

$m = -1, \frac{1}{2} \pm 0.866i$

$CF = C_1 e^{-x} + e^{1/2 x}(C_2 \cos 0.866x + C_3 \sin(0.866x))$

P.I : $\left.\frac{e^x z}{D''^3 + 1}\right|_{D = D+1}$

$\frac{e^x z}{(D+1)^3 + 1}$

$e^x \left(\frac{z}{(D+1)^3 + 1}\right)$ } poly

$e^x \left(\frac{z}{D^3 + 1 + 3D(D+1) + 1}\right)$

$e^x \left(\frac{z}{D^3 + 2 + 3D^2 + 3D}\right)$

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$= 1/2 - 3/4$

$1 + 3D + 3D^2 + D^3$ | z

$\pm z \quad \pm 3/2$

$0 \quad -3/2$

$-3/2$

$+$

0

$e^z \left(\frac{z}{2} - \frac{3}{4} \right)$

$PI = x \left(\frac{\log x}{2} - \frac{3}{4} \right)$

$y = CF + P.I$